

### Assignment: vectors

- Two vectors, both equal in magnitude, have their resultant equal in magnitude to either. Find the angle between the two vectors. [ $120^\circ$ ]
- The greatest and the least resultant of two forces acting at a point are 29 N and 5 N respectively. If each force is increased by 3 N, find the resultant of the two new forces now acting at right angles to each other.
- The sum of the magnitudes of two forces acting at a point is 18 N and the magnitude of their resultant is 12 N. If the resultant makes an angle  $90^\circ$  with the force of smaller magnitude, what are the magnitudes of the two forces?
- A particle has a displacement of 12m towards east and 5 m towards north and then 6m vertically upwards. Find the net displacement.
- A motor boat is racing towards north at 25 kmph and the water current in that region is 10 kmph in the direction of  $60^\circ$  east of south. Find the resultant velocity of the boat.
- On a certain day, rain was falling vertically with a speed of 35 m/s. A wind started blowing with a speed of 12 m/s from east to west. In which direction should a boy waiting at a bus stop hold his umbrella?
- A man can swim with a speed of 4 kmph in still water.
- Rain is falling vertically with a speed of  $30 \text{ m s}^{-1}$ . A woman rides a bicycle with a speed of  $10 \text{ m s}^{-1}$  in the north to south direction. What is the direction in which she should hold her umbrella?
- How long does he take to cross a river 1.0 km wide if the river flows steadily at 3.0 km/h and he makes his strokes normal to the river current? How far down the river does he go when he reaches the other bank?
- In a harbour, wind is blowing at the speed of 72 km/h and the flag on the mast of a boat anchored in the harbour flutters along the N-E direction. If the boat starts moving at a speed of 51 km/h to the north, what is the direction of the flag on the mast of the boat?
- A river 800m wide flows at the rate of 5 kmph. A swimmer who can swim at 10 kmph in still water, wishes to cross the river straight. (i) Along what direction must he strike? (ii) What should be his resultant velocity? (iii) How much time he would take to cross?
- A particle is acted upon by four forces simultaneously: (i) 30 N due east (ii) 20 N due north (iii) 50 N due west and (iv) 40 N due south. Find the resultant force acting on the particle.
- Find the vector AB and its magnitude if it has initial point A (1,2, -1) and final point B (3,2,2)
- Find a unit vector parallel to the resultant of the vectors  $A = i + 4j - 2k$  and  $B = 3i - 5j + k$
- Two forces  $F_1 = 3i + 4j$  and  $F_2 = 3j + 4k$  are acting simultaneously at a point. What is the magnitude of the resultant force?
- If the magnitude of two forces are 3 and 4 and their scalar product is 6, then find the angle between them.
- Prove that the vectors  $A = i + 2j + 3k$  and  $B = 2i - j$  are perpendicular to each other.
- If  $\vec{A} + \vec{B} = \vec{C}$  and  $A^2 + B^2 = C^2$ , then prove that  $\vec{A}$  and  $\vec{B}$  are perpendicular to each other
- If unit vectors  $\hat{a}$  and  $\hat{b}$  are inclined at angle  $\theta$ , then prove that  $|\hat{a} - \hat{b}| = 2 \sin \frac{\theta}{2}$
- Prove that:  $(\vec{A} + 2\vec{B}) \cdot (2\vec{A} - 3\vec{B}) = 2A^2 + AB \cos \theta - 6B^2$
- If A and B denote the sides of a parallelogram, and its area is  $AB/2$ , find the angle between A and B

22. For any two vectors A and B, prove that  $|\vec{A} \times \vec{B}|^2 = A^2 B^2 - (\vec{A} \cdot \vec{B})^2$

23. Prove that  $((\vec{a} + \vec{b}) \times (\vec{a} - \vec{b})) = 2(\vec{b} \times \vec{a})$